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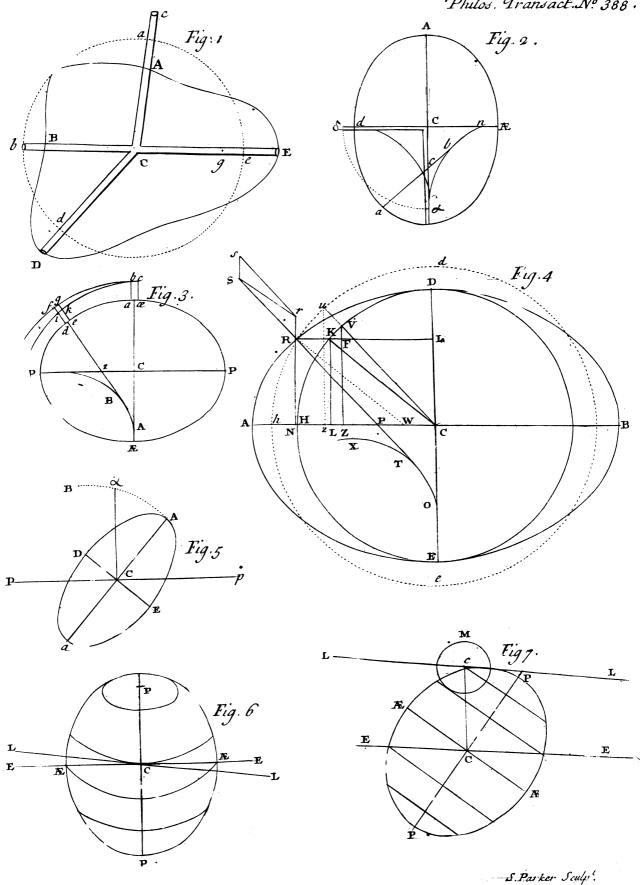
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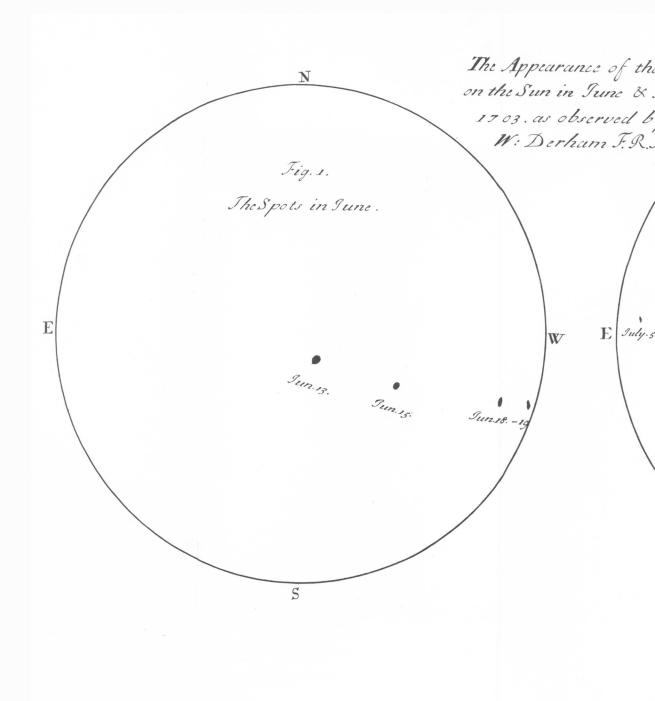
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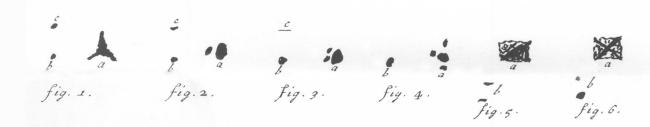
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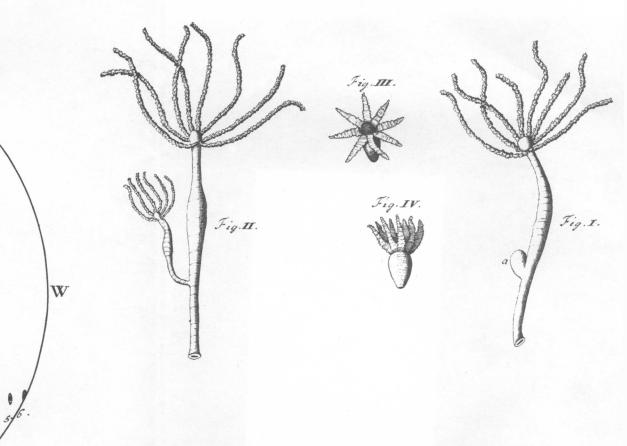
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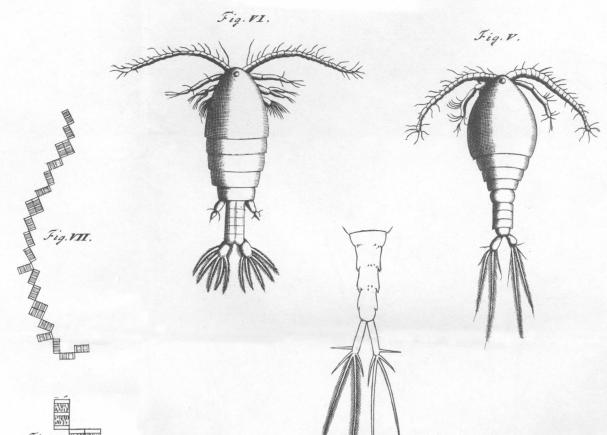


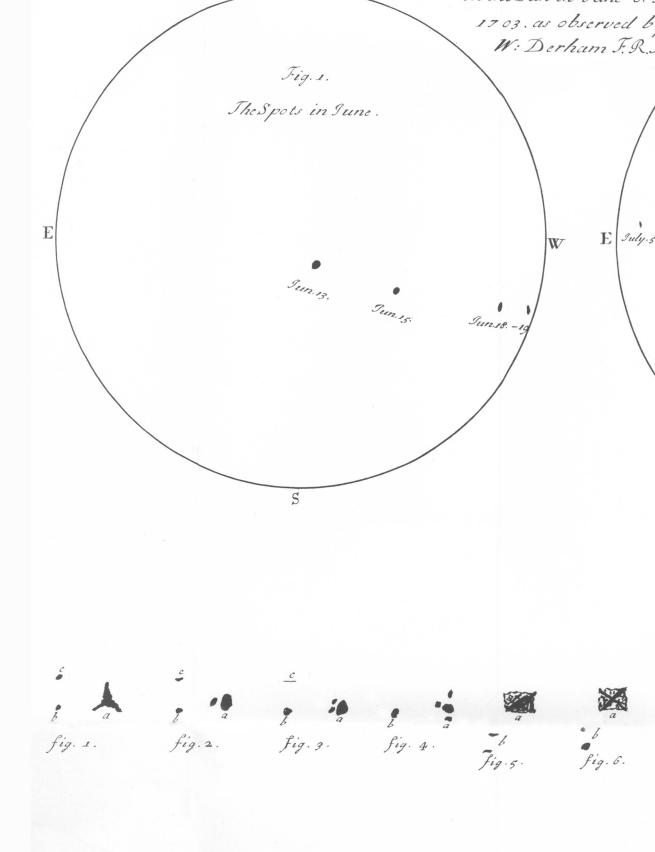


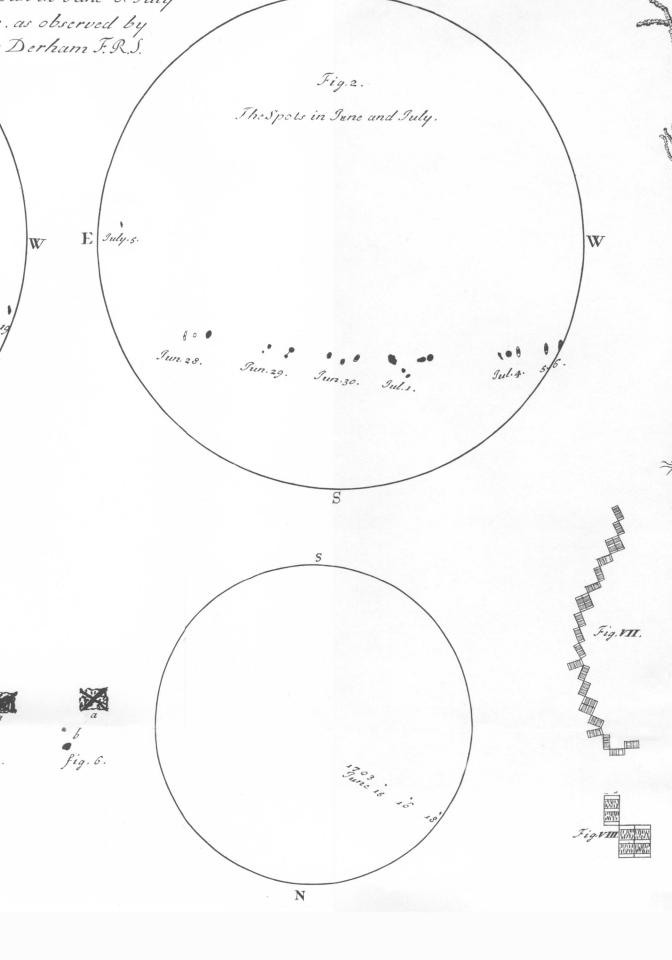


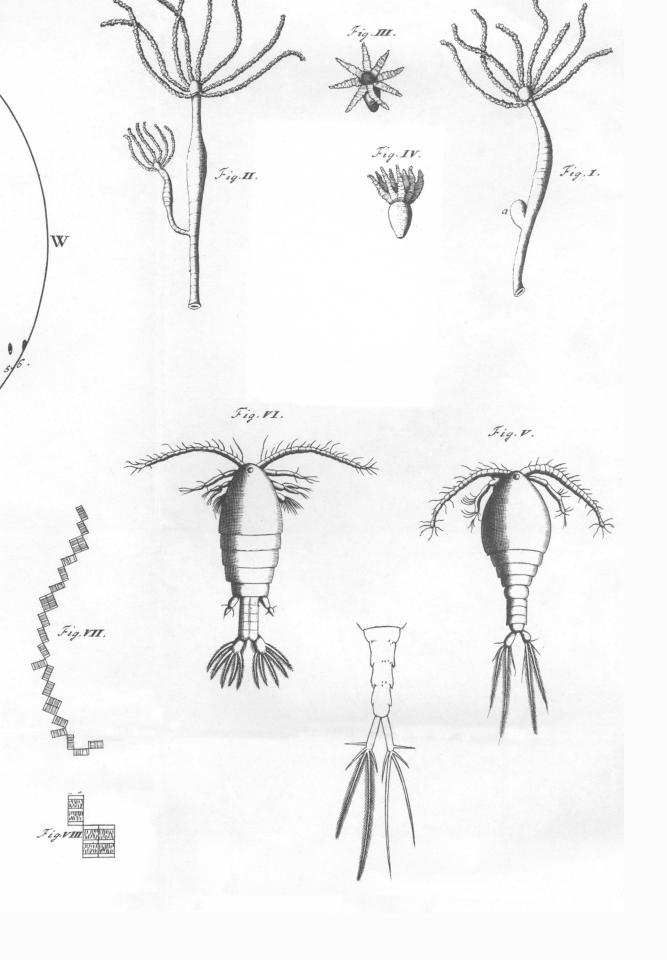
ppearance of the Spots J Sun in Trunc & July . as observed by Derham F.R.S. Fig. 2. The Spots in June and July . E July . 5. W Jun. 28. Sun. 29. Jul.4. 1303 is











I. A Dissertation concerning the Figure of the Earth. Part the Second. By the Reverend J. T. Desaguliers, LL. D. F. R. S.

CINCE my * Paper concerning the Figure of the Earth was read before the Royal Society, I met with a Differtation of Monf. Mairan (in the Memoirs of the Royal Academy of Paris, for the Year 1720.) wherein the learned and ingenious Author has taken a great deal of Pains to reconcile the Observations made on Pendulums, (found to be shorter at the Æguator than at Paris, when they swing Seconds) with the oblong spheroidical Figure of the Earth, deduced from Monf. Castini's Measures. And tho' upon a strict Examination of his Conjectures, and what he gives for Demonstrations. I do not find Reason to alter my Opinion concerning the oblate or flatted Spheroid, which Sir Isaac Newton has shewn to be the Figure of the Earth; yet fince it might be thought by some, who have read Mons. Mairan's Treatife, and afterwards may read mine, that I have not consider'd all the Circumstances that He has done, and that I have not been exact enough in the Mathematical Part of my Differtation, because I have drawn fome Conclusions from supposing the Figure of the Earth fpherical, when I should have suppos'd it an oblong Spheroid; I beg Leave to shew here, wherein I think Monf. Mairan is mistaken, and to give those additional Proofs of my Assertions, which I promis'd the Society when I gave in my last Paper.

^{*} Vid. Philof. Trans. No. 386, 387.

First then I begin with the Conjectures.

Monf. Mairan says, that it is as reasonable to suppose the Earth (if it was once shuid) to have been an oblong Spheroid at first, as a Sphere; and that, in such a Case, the Centrisugal Force of the several Parts of the Earth, arising from its Revolution about its Axis, which might convert a Sphere into an oblate Spheroid, wou'd only

change an oblong Spheroid into one less oblong.

If the Earth was at first a Fluid, (supposed homogeneous, and of any given Form,) and left to those Laws, which we find to obtain at present; it must put on a spherical Figure, for the same Reason that Drops of Mercury, of Water, and other Fluids, put on such a Figure. And to suppose any Change made in that Figure from the Pressure of an external Fluid, filling up all Space, is contrary to what has been demonstrated by Sir Isaac Newton in his Principia Lib. 2. Prop. 19. where he shews, That if any Portion of a Fluid be compress'd by the same or any other homogeneous Fluid, that Portion will not have its Figure alter'd by that Pressure.

And indeed we see, that in the Receiver of the Air-Pump, Lumps of Butter, coagulated Oil, or Honey, Drops of Quicksilver or Water, &c. have the same Figure, whether the Pressure of the Air acts upon them,

or be taken off by exhausting the Receiver.

That a fluid Substance, of any Figure, will by the Gravity of its Parts become spherical, is plain by the following

DEMONSTRATION. Fig. I,

Let ABCDE be a Portion of an homogeneous Fluid, whose Parts tend towards one another, and whose Figure

is not spherical. If in such a Fluid we suppose a Syphon as ACE (or which is the same thing, if all the Fluid shou'd be frozen, except the Canal ACE) whose Legs AC and CE are unequal, and meet at C, the Center of the Fluid, towards which there is the greatest Tendency; the Fluid will run out at A in the Leg AC, till it be come down as far as g in the Leg CE, supposing Cg equal to AC. But if the Leg AC be lengthened as far as c, then the Fluid will only come down as far as e in the Leg CE, and at the same time rise up to a in the Leg Ca, Ca being equal to Ce.

If fuch another Canal or Syphon be suppos'd at BCD, the Fluid in it will come down from D to d, and rise from B to b. And since such Syphons may be suppos'd all over the Fluid ABDE; that Fluid, by the mutual Tendency of its Parts towards one another, must be reduc'd to the spherical Figure abde. Which was to

be demonstrated.

Now, without confidering the Unreasonableness of the Supposition, let us imagine the Earth to have been an oblong Spheroid at first, and then to have a diurnal Revolution given to it, which shou'd by Degrees shorten its Axis, to bring it to what Messieurs Cassini and Mairan suppose it at present to be. If in such a Case the Earth be suppos'd fluid enough to change its Figure. by the Revolution about its Axis, why should it stop when the Equatorial Diameter comes to want just one 96th Part of the Length of the Axis? fince two Powers act upon it to shorten its Axis, viz. Gravity, and the Centrifugal Force; the first of which has already been shewn capable to reduce it to a Sphere, and the Centrifugal Force is acknowledged by Monf. Mairan to be (as Sir Isaac Newton has prov'd it) at the Æquator equal to ? Part of the Gravity there. Certainly the Alteration of Figure wou'd not have stopp'd, before the Rr2 Earth

Earth came to be a Sphere; nay, and it must have rifen at the Æquator; and how much, I have already

shewn in my former Paper.

Again, if we suppose the Earth of an heterogeneous Fluid, before the diurnal Revolution, the heaviest Parts wou'd go towards the Center, and the lighter towards the Surface; and that Way the Terraqueous Globe wou'd also become a Sphere. Then if, when the Central Parts are fix'd, and the superficial Strata are still fluid, the Earth receives a diurnal Motion; it will rife at the Æquatorial Parts, and that to a greater Height than what I have shewn in my former Paper, where I fuppos'd the Earth of uniform Matter. And that fomething like this must be the Case, appears from what Sir Isaac Newton has said upon this Subject. For after having shewn, from supposing the Earth of uniform Matter, that the Centrifugal Force of all its Parts wou'd bring it to be 17% English Miles higher at the Æquator than at the Poles, and after having given a Table of the proportionable Decrease of the Length of the Degrees of a Meridian of the Earth, going from the Poles to the Æquator, in fuch a Figure of the Earth, with the Lengths that Pendulums must have to swing Seconds in feveral Latitudes; from a Comparison of the Lengths of Pendulums (observ'd by different Persons to be shorter towards the Æquator, than in greater Latitudes (when they fwing Seconds) he shews that the Earth must be 313 Miles higher at the Æquator than at the Poles; and therefore that it must be denser towards the Central than the Superficial Parts to produce a flatted Spheroid, where the Æquatorial Diameter must exceed the Axis so much more; that is, be longer fomething more than Tas Part.

Laftly, let us suppose the Earth, at its first Creation, to have been made of Land and Water, the first as solid,

and the last as stuid as it is now, but of Mons. Cassini's Figure, and examine the Consequence. Since in that Figure the Axis is for Part longer than the Æquatorial Diameter; the Gravity will be so much greater at the Æquator than at the Poles, that the Waters will all flow to the Æquatorial, and leave the Polar Regions; which will happen still more by the Centrifugal Force, which the Earth in its diurnal Motion will give to the Fluid; and therefore the Sea wou'd be 43 for Miles (reckoning 5000 Feet to a Mile) higher at the Æquator than at the Poles, which must overslow all the Torrid Zone, and leave the Polar Regions dry.

I am very well aware, that it may be objected by fuch as have read Monf. Mairan's Differtation, and have not read Sir Isaac Newton's Principia, or have not read that Book with due Attention _ " That I have not ar-"gued fairly in drawing Consequences from a greater "Gravity at the Æquator than at the Poles, in an ob-" long Spheroid; because Mons. Mairan has shewn. " that, in such a Figure of the Earth, the Gravity is greater " at the Poles than at the Æquator; and that I shou'd " have drawn my Consequences from these Principles." To which I answer, that his Demonstrations about Gravity are built upon wrong Suppositions, as I shall shew by Nevertheless, supposing that Gravity was greater towards the Poles than towards the Æquator, in the Proportion that he assigns, namely of the Ray of Curvature drawn into the Perpendicular to the Curve, terminated at the Axis; let us consider what will follow from his Principles.

Let us then suppose the Earth at first in a suid State; AA the Axis, dA the Æquatorial Diameter, aba Ray of Curvature, dn another, ac and dC two Lines of Tendency or Perpendiculars to the Curve, intercepted by the Axis at c and C; and dC, AC, two Tubes or Canals of the Fluid, gravitating towards, and communicating at C. I fay that, according to Monf. Mairan's Principles of Gravity, the Earth cannot preserve its oblong spheroidical Figure. For since the Gravity at a : Is to the Gravity at $d:: As dn \times dC: to ab \times ac$, it will follow (from the Nature of the Ellipse) that the Gravity at A: will be to the Gravity at $d:: As A C^4$: to dC: and therefore the Forces, with which the Columns of Fluid A C and d C tend towards C, will be as their Masses drawn into the Forces driving towards C, that is, as $A C \times A C^4$ to $d C \times d C^4$. Now by the Principles of Hydrostaticks, it is evident that the Fluid, in the Canal A.C., will cause the Fluid in the Canal d.C. to run out at d as long as $AC \times AC^{2}$ is greater than $d C \times d C^4$: And if the Canal C d be continued quite to δ , the Surface of the Fluid in AC will fink to α , whilst the Surface of the Fluid in d Crises up to S, in which Case as $\alpha C = C \beta$, the Point A will come to a, and the Point d to s, and the Curve A d being chang'd into a d, the oblong Spheroid will be chang'd into a Sphere, the only Figure confistent with the *Aguili*brium of the fluid Parts, according to Monf. Mairan's own Principles; because then you will have A C4 = $d C^4$, and $A C \times A C^4 = d C \times d C^4$. If we make use of Sir Isaac Newton's Principles in this Reasoning, we shall also shew, that an oblong, spheroidical, sluid Earth will be chang'd into a Sphere; but not fo fast as it does by Monf. Mairan's Laws; for, according to Sir Is to the Gravity at A: Is to the Gravity at $d:: As \sqrt[4]{C}d: \sqrt[4]{A}C. \quad \mathcal{Q}. \quad E. \mathcal{D}.$

NB. Here we have suppos'd no diurnal Revolution, for as soon as that begins, the Centrifugal Force will raise the Æquatorial Parts, and change the Sphere into a flatted Spheroid, as has been before shewn, and is allow'd by Mons. Mairan.

Now if we suppose the same Figure of the Earth, but the Land (at its first Creation) as firm as it is now; it will in that Case follow from Mons. Mairan's Principles, that the Sea must rise and overflow all the Æquatorial Regions, tho' the Earth had no diurnal Revolution; and much more so, when the Centrifugal Force, arising from the diurnal Motion, helps to carry the Water the same Way.

DEMONSTRATION.

Let P & P Æ * represent the Plane of a Meridian, P P the Axis of the Earth (suppos'd an oblong Spheroid) & Æ the Diameter of the Æquator, de a & Part of the Surface of the Earth, & A and e B two Perpendiculars to the Surface of the Earth (which are here two Rays of Curvature) fc the Surface of the Sea, and fdeg, bacc two Cylinders of Sea-Water of equal Bases and equal Heights.

Since Gravity acts on the two equal Columns of Water baca, fdeg in the reciprocal Ratio of the Ray of Curvature (at the respective Places of the Columns) drawn into that Part of it which Mons. *Mairan* calls the Line of Tendency, (that is, in the Ratio of $eB \times eZ$ to $eA \times eC$) the Weight of fe: will be to the Weight of ba: As $eA \times eC$: to $eB \times eZ$. Therefore if there be a Communication between the fluid Co-

NB. That the Centrifugal Force will still add to the Height of the Sea at bc, is plain from what we have said before. And if we apply these Principles to determine the different Lengths of Pendulums, swinging Seconds at Paris and at the Aquator; from the Gravity at Paris, compar'd to the Gravity at the Aquator (in this Supposition of the Action of Gravity and Figure of the Earth) a Pendulum must be shorter at the Aquator by more than 10 Lines, without considering the Centrifugal Force; and if the Centrifugal Force be taken into Consideration, the Pendulums must be shortened near a whole Inch. But this being about five Times more than agrees with Observation; what proves too much, proves nothing at all.

Having thus shewn, that Mons. Mairan's Account of the Action of Gravity, on several Places upon the Earth's Surface, can be of no Service for reconciling the Experiments made on Pendulums, with the Figure of the Earth deduc'd from Mons. Cassini's Measures: I proceed to shew that his Demonstrations are founded upon wrong Principles. And first, in Relation to Gravity.

This Gentleman has follow'd Sir Isaac Newton, in faying, that Gravity increases in a duplicate reciprocal

Proportion of the diminish'd Distance from the Center of the Force, and so vice versa; but he has follow'd Sir Isaac Newton no farther than ferv'd his present Purpose: otherwise he wou'd have known.—That in respect to a Central Body (as a Planet) towards which others are (attracted or) impell'd by Gravity, this Law obtains only as Bodies attracted, are remov'd from the Surface of the Planer, to greater Distances from the Center compar'd with that Distance; or as from greater Distances they approach nearer to the Planet. - That the greatest Action of Gravity is at the Surface of the Planet.—That afterwards in advancing towards the Center. the Force of Gravity, on the Body attracted, continually grows less, decreasing directly as the Distance; and that this holds true in a Spheroid as well as a Sphere. — That on different Parts of the Surface of the Earth (in the Condition it is now) the Gravity on Bodies is reciprocally as their Distance from the Center of the Earth. — That though at a confiderable Diffance we look upon the Earth, or any Planet, or even the Sun, as a Point (in the Center of the Forces tending towards it) endued with an absolute Force, proportional to its Quantity of Matter; yet when we come so near the Body as to consider the Space it takes up, we are to take notice. that the whole Attraction or Gravity of the Body, is made up of the Sum of the Attraction of all its Parts properly combin'd; and therefore, that when a Corpufcle, or Body attracted, comes to be within the Planet, or Body attracting, the Matter above it draws it back in fuch a Manner, that it leaves it only a Force to go on towards the Center, which is directly as the Diffance, as we have already faid; just as if a Body concentric to the Planet (whether spherical or spheroidical) had its Surface just where the Corpuscle is, and all the exterior Crust or Shell was annihilated.

I do not doubt but Mons. Mairan will be of this Opinion, when he has carefully and impartially examin'd the 12th and 13th Sections of the First Book of Sir I-faac Newton's Principia, and the 18th, 19th, and 20th Prop. of the Third Book. And if he will be at the Pains to compare the 38th and 39th Proposition of the Third Book with the 66th of the First, he will find that the Precession of the Acquinoxes is owing to the broad spheroidical Figure of the Earth; and that if it had Mons. Cassini's Figure, the Acquinoctial Points wou'd move in Consequentia faster than they do now in Antecedentia.

Further, Mons. Mairan demonstrates, that in an oblong Spheroid, the Diminution of Gravity, by the Centrifugal Force, encreases faster in going from the Poles to the Æquator, than it wou'd do in a Sphere, and faster in a Sphere than it wou'd do in a broad Spheroid; and therefore wou'd shew, "That notwithstanding the Sur-"face of the Earth is nearer to the Center in Mons. Case simi's Figure than in Sir Isaac Newton's, yet the Centrifugal Force will diminish the Gravity so fast in going from Paris to the Æquator, that the shortening of Pendulums, to make them swing Seconds at the Æquator, may very well be accounted for that Way." Now let us examine into this Matter, to see whether the Cause is adequate to the Effect.

If the Distance from the Surface of the Earth at the Pole to the Center be 96, and the Distance of the Surface at the Æquator be 95, the Distance of the Surface at Paris, in the Latitude of 48° 50′, will be 95,562, &c. by the Property of the Ellipse. Now since the Force of Gravity, in different Places on the Earth's Surface, is reciprocally as the Distance from the Center, and the Lengths of Pendulums, that perform their Vibrations in the same Time, are directly as the Force of Gravity; therefore

therefore the Length of Pendulums at Paris, will be to their Length at the Equator, as 95 to 95,562, &c. that is, as 440,555, &c. to 443,165, &c. and consequently they must be lengthen'd 2,61 &c. Lines. But as from Monf. Mairan's Principles, the Diminution of Gravity by the Centrifugal Force, is greater at the Æquator than at Paris, hardly 4 Part of the whole Gravity at the Æquator, the Pendulums must be shortened in that Proportion; fo that then the Length of a Second-Pendulum, will be 440,555 + 2,61 — I Lines. But as that Quantity is greater than 440,555, &c. therefore the Pendulums upon the Whole must be lengthen'd: Nay, though we shou'd allow a shortening of two Lines; since by Obfervation Pendulums are found to be about two Lines shorter at the Æquator, the oblong spheroidical Figure of the Earth cannot be consistent with the Experiments on Pendulums.

I beg Leave to set down Mons. Mairan's aforesaid Demonstration here; that we may see whether he has assum'd true Principles.

PROPOSITION V.

"*XI. The Centrifugal Force at any Degree of Latitude, taken upon the oblong Spheroid, between the Æquator and the Pole, is less in Comparison to the Centrifugal Force at the Æquator, than it wou'd be at the same Degree of Latitude taken upon a Sphere; or, which is the same thing, the Centrifugal Force encreases more, going from the Poles towards the Æquator, upon an oblong Spheroid, than upon a perfect Sphere; and consequently Grawity diminishes more, and a Pendulum must be more

^{*} See Monf. Mairan's Differtation, Article XI. &c.

" Shortened under the Æquator, in the Hypothesis of the oblong Spheroid, than in that of a perfect "Sphere."

"Sphere.
"Having describ'd an oval Curve of any Kind, as for Example, the Ellipse * ADBE abovementioned, and inscrib'd the Circle DHE, whose Radius is DC = half the shorter Axis DE; upon AD take any Point as R, between the Æquator and the Pole, and from that Point to the Evoluta OTX draw the Ray of Curvature RT, which gives the Line of Tendency RP (Art. IV.) Draw likewise from the common Center C, to the Circumference of the Circle DH, a Radius CV, parallel to PR, and meeting the Circle at V; then from the Points R, V, draw the Lines RN, VZ, perpendicular to the Axis AB.

"It must be observ'd, First, That as the Ellipse AD represents a Meridian of the oblong Spheroid, the Circle DH represents a Meridian of a Sphere in the fame Plane.

- "Secondly, That the Point V, on the Circular Meridian, answers to the same Degree of Latitude as
 the Point R, upon the elliptical Meridian; because
 the Lines PR, CV, being parallel to each other, and
 perpendicular, the one to the Ellipse and the other
 to the Circle (by Construction) the touching Planes,
 or Horizons of the Points R, V, will also be parallel.
- "Thirdly, Whence it follows that the Diminution of the Centrifugal Force (acting against Gravity) on account of its Obliquity to the Horizon (Art. X.) of

" the same Degree of Latitude on the Elliptical and on " the Circular Meridian, is the same in both Cases, and " in the same Ratio as the absolute Centrifugal Forces " represented by the Perpendiculars R N, V Z, (Art. IX.) "Therefore to know whether the Centrifugal Force " (whether absolute or relative) of the Point R, upon the " oblong Spheroid ADBE, be less or greater in respect " to the Centrifugal Force under the common Æquator " DE, than the Centrifugal Force (whether absolute or relative) of the correspondent PointVupon the Sphere: " nothing more is requir'd than to fee which is the " longest of the two Perpendiculars, namely, R N in "the oblong Spheroid, or VZ in the Sphere; fince " these two Lines express the Radii of the Circles of « Revolution, and confequently the absolute Quantity " of the Centrifugal Forces.

" 4thly and lastly, That the Ratio of the Centrifugal " Forces of two correspondent Points upon the oblong " Spheroid A DBE, and the inscrib'd Sphere DHE, to the " Centrifugal Force of their Æquators is the same, suppo-" fing the Sphere of any other Bigness; and that it has " been determin'd here of the Diameter DE, only to " render the Demonstration easier, by giving the same " Consequent to the Antecedents R N and VZ. For " if about the Center C and with the Radius Cd, the " Circle dhe be describ'd equal (for Example) to a " Meridian of a Sphere of the same Solidity as the obo long Spheroid ADBE; and the Radius CV be pro-" duc'd till it meet the Circle db at the Point u, and " uz be let fall perpendicular to the common Axis of Revolution, and parallel to VZ: It is plain, that " we shall always have V Z: D C:: uz: dC, or " $\frac{VZ}{DC} = \frac{uz}{dC}$, and consequently $\frac{RN}{DC}$ will have the

" fame Ratio to $\frac{VZ}{DC}$ as to $\frac{uZ}{dC}$.

"Therefore, in order to demonstrate that the Centri"fugal Force of a Point, taken in any Latitude upon
"the oblong Spheroid, is less when compar'd to the Cen"trifugal Force of the like Point, taken upon a Sphere
"in respect to the Centrifugal Force at the Æquator;
there is nothing more requir'd than to shew that R N

" $\langle VZ \rangle$, because by that means we shall have $\frac{RN}{DC}$ " $\langle VZ \rangle$

"This being observed; from the Point R, draw the Line RI, parallel to the Axis AB, and meeting the "Circle DH at K, and the Diameter DE of the Æquator at the Point I. From the Point K having let fall the Perpendicular KL = RN, upon the Axis AB, and drawn KC to the Center C; the Question will be brought to this, viz. To know whether the Point V coincides with the Point K; or whether it is above it towards D, or below towards H.

"But CK = CV = CD > PR (Art. VIII.) therefore CK and PR being both between the Parallels
AC, RI, the greatest CK is more inclin'd to them
than the least PR, and the Angle KCA is less than
the Angle RPA = VCA. And since these two
Angles have each of them one of their Sides coinciding
with the Line AC, namely, the Side AP of the Angle
RPA, and the Side AC of the Angle KCA, it follows that the Side VC of the Angle VCA = RPA
> KCA, will go above CK between CK and CD,
and meet the Line RI at the Point G, between K and
I, and the Circle DH at the Point V, which consequently

" quently will be above RI, between K and D. There" fore CV = CG + GV is = PR + GV, and con" fequently VZ, which meets RI at the Point F, is
" = ZF + FV = RN + FV; and therefore RN
" = VZ - FV. Therefore RN < VZ.

"And because the same Thing may be demonstrated in respect of any other Point, taken between the Acquator and the Pole; and that Gravity, and consequently the Length of a Pendulum diminishes, as the Centrifugal Force encreases. Therefore &c. Q. E. D.

COROLLARY.

"XII. From what has been demonstrated, and from Prop. 3. Art. VIII. it follows, that the Perpendicular which is drawn from any Point of an oval Meridian to the Axis, will be so much shorter, in Comparison to the Perpendicular drawn from the correspondent Point of an inscrib'd circular Meridian, as the Latitude is greater; and consequently (by Art. XI. Num. 3.) the Centrifugal Force will be so much the less, and Gravity so much the greater, upon the oblong Spheroid, in respect to the Centrifugal Force, and the Gravity under the Æquator.

"For as the Line RP does always decrease, as the Point R is taken nearer to the Pole A, it is evident, that the Angle VCK will continually encrease, in respect to the Angles VCA, KCA, as it is their Difference, and consequently that the Perpendicular VZ will be so much greater than the Perpendicular KL = RN.

I pass over the Demonstration of the latter Part of his Proposition above-mentioned, which he deduces justly from his Construction, if what he says (Num. 2.) be right; because in such a Case it cannot be call'd in Question:

ftion; and proceed to an Observation that he makes afterwards, viz, " We must take care to observe in the " foregoing Propositions and Corollaries, that the "Comparison is always made between two similar " Points of Latitude, taken upon the two Spheroids, " or upon one of the Spheroids and the Sphere. " between the Aguator and the Poles, in respect to " the Centrifugal Force upon the Æquator of any " one of these Spheroids, or of the Sphere. For if " we only compar'd absolutely the Centrifugal Force " of a Point of the Aguator of the one, to the Cen-" trifugal Force of a correspondent Point of the A. " quator of the other, it is plain that it wou'd be " greater upon a flatted Spheroid than upon a Sphere. " or than upon an oblong Spheroid of the same Soli-" dity, in the Ratio of the great Axis of the gene-" rating Ellipse of the flatted Spheroid, to the Di-" ameter of the Sphere, or to the shorter Axis of the " generating Ellipse of the oblong Spheroid. And in " all Likelihood, this must be the Reason that has " made others, who have treated of this Subject, to " imagine the very contrary of what I have demon-" Arated.

As Mons. Mairan considers the Earth at rest, in the Construction for his Demonstration above quoted, and afterwards observes what Effect the Centrifugal Force will have upon Bodies on its Surface, to diminish the Gravity, with which they endeavour to descend in their Line of Tendency RP: He shou'd not only have taken notice (as he has done) that the whole Centrifugal Force NR is not to be substracted from the Gravity at R, as the whole Centrifugal Force CD is to be substracted from the whole Gravity at D, because of the Obliquity of RN to PR; but he should have observed also, that the Obliquity of the Plane of the Parallel NR, in which

the Centrifugal Force acts, must alter the Line of Tendency RP, and change the Direction RP into RW, fomewhere between the Point P and the Center C; for if there be a heavy Body as a Plummet, hanging by a Thread in the Line SR, or SP, the Line of Tendency which has been suppos'd perpendicular to the Curve A R D, without taking in the Effect of the Centrifugal Force; as foon as the Spheroid revolves about its Axis. the Body which wou'd fall in the Line SR, acted upon only by one Force, namely, that of Gravity, will now be acted upon by another Force, at the fame Time pushing it in the Line Ss (which is the fame as Rr) and confequently will move in the Line Sr, diagonal of the Parailelogram s S R r; or, which is all one, a Body plac'd at R will have its Line of Tendency in RW, as I have already shewn in my first Differtation on this Subject; only I did not suppose the Earth a Spheroid before the diurnal Motion, and therefore made use of the Line ZV instead of the Line NR; so that it may be objected that the Angle rSR will not be so great in a Spheroid as in a Sphere, because the Centrifugal Force which acts with the same Obliquity (since NRP = ZVC) is as much less in the Spheroid as NR is less than ZV: But I was aware of that, and therefore made the Angle R Sr only of 5 Minutes, when it really appears to be of almost fix Minutes, when the Earth is suppos'd spherical; and therefore, without coming to give the exact Quantity of the said Angle, one may easily perceive, that Monf. Cassini's Difference of the Axis and Æquatorial Diameter will produce a Figure, in which the Angle R Sr, will not be less than of 5 Minutes.

Such an Obliquity, caus'd in the Direction of Gravity, will render the oblong spheroidical Figure of the Earth impossible, because then Fluids would not have the Lines of their Gravity perpendicular to the Horizons of the Places

Vol. XXXIII. Tt Places where they are, (Supposing the Horizons of Places to be Planes touching the Curve of the Earth in those Places) and Plumb Lines wou'd be fo far out of the Perpendicular to Lines of Level, as to make an Angle eafy to be observed, as I have shewn in my former Paper.

But if the fame Cause be supposed to act upon the Sea to make it level, as makes heavy Bodies to fall (which certainly must) then indeed Lines of Level will be perpendicular to Plumb Lines, and the Level of the Sea, taken always for the Horizon of a Place, will not be a Plane touching the Earth, but cutting it towards the Poles, and consequently the Water will be carried towards the Æquator, as was before shewn.

Besides, the Difference of the Action of the Centrifugal Force wou'd not be so great between correspondent Points of the same Latitude in the Spheroid and in the Sphere; for when the Line of Tendency RP is by the Centrifugal Force chang'd into R W, the Point R upon the Spheroid does no longer correspond in Latitude with the Point V upon the Sphere, but must be taken nearer to V, so that the Line RW may become parallel to VC, and RWA = VCA.

If it be alledged here, that Monf. Mairan supposes the Earth in Motion, and takes in the Effect of the Centrifugal Force, when he makes the Line of Tendency to be RP; I answer, that if he had consider'd the Earth as revolving upon its Axis, he wou'd not have made V C the Line of Tendency of a spherical Earth in Motion, fince it is the Line of Tendency of such an Earth at

In Monf. Mairan's Observation above-mentioned, he fays, " that we are not to compare the Centrifugal Force at the Æquator of an oblong Spheroid, with the Centrifugal Force at the Æquator of a Sphere, or at the **Æ**quator "Acquator of a flatted Spheroid of the same Solidity; allowing that then it wou'd be greater in the Sphere, and still greater in the flatted Spheroid: but only the Centrifugal Forces in several Latitudes upon the same Figure." —But I beg Leave to differ from him for the following Reasons.

First, Because the Force of Gravity is not the same at the Æquator of the stated Spheroid, as it is at the Æquator of the Sphere, or as it is at the Æquator of the oblong Spheroid.

Secondly, Because it is not the same in different Latitudes, in either of the Spheroids. (See Sir Isaac Newton Lib. 3. Prop. 19 and 20.) And Mons. Mairan's Way of arguing will only serve, in Case the Gravity shou'd be the same in all the Points of the Surface of the Earth in his Figure, and also in the two other Figures.

For Example, let the uniform Gravity be call'd g; and Firft, let the Centrifugal Force at the Æquator of the flatted Spheroid be call'd c + 2; and the Centrifugal Force in any Latitude, as for Example, the Latitude of Paris (as it is diminished on Account of a shorter Co-sine of Latitude, and likewise on Account of its Obliquity to the Line of Tendency,) be call'd c + 2 - l; the Difference of the Diminution of Gravity at Paris, and at the Æquator will be g-c+2-g-c+2-l.

Secondly, Let the Centrifugal Force at the Equator of the Sphere be call'd c + 1, and the Centrifugal Force at the Latitude of Paris be call'd c + 1 - 1 + m; the Difference of the Diminution of Gravity at T t 2 Paris,

Paris and at the Equator in a spherical Earth, will be g-c+1-g-c+1-l+m=l+m.

Thirdly, Let the Centrifugal Force at the Æquator of the oblong Spheroid be call'd c, and the Centrifugal Force at Paris be call'd c - l + m + n; the Difference of the Diminution of Gravity at Paris, and at the Æquator, in an oblong spheroidical Earth, will be g - c - l

$$\sqrt{g-c-l+m+n}=l+m+n.$$

Now, if Gravity shou'd in every Case be equal to g, it is evident, that the shortening of Pendulums, at the Equator, wou'd be greater in the oblong Spheroid, than in the Sphere, or in the flatted Spheroid; because as the Lengths of Pendulums diminish with the Gravity, those Lengths will be at Paris and at the Equator, when compar'd, as g-c+1-l to g-c+1 in the flatted Spheroid; as g-c+1-l+m to g-c+1 in the Sphere, and as g-c-l+m+n to g-c in the oblong Spheroid; and consequently, from what M. Mairon has demonstrated this Ratio of g-c-l+m+n to g-c, being greater than either of the others, the Pendulums must be shortened in the oblong Spheroid.

But as the Force of Gravity is less at the Æquator of the flatted Spheroid, than at the Æquator of the Sphere, or of the oblong Spheroid of the same Solidity: let us express its Quantity in the three Cases by g - s, g, and g + s, and we shall then find the Lengths of the Pendulums, at the Æquator of the three Solids, as g - s - c + 2, g - c + 1, and g + s - c; consequently the Lengths of Pendulums

Pendulums will be greatest at the Æquator of the oblong Spheroid, because g + s - c is the greatest Quantity.

Lastly, To compare the Lengths of Pendulums at the Alguator of the oblong Spheroid, thus found, with their Lengths at the Latitude of Paris upon the faid Spheroid — Let us express the Excess of Gravity at the Æquator, whereby it is greater than at Paris (because in this Figure, Paris is farther from the Center of the Earth, than the Æquator, by Fig Part) by the Letter s. and the Excels of the Centrifugal Force at the Aguator, above that Part of it which acts directly against Gravity at Paris, by l + m + n, the Gravity at Paris by g, and the Centrifugal Force at the Æquator by c; then g + s - c will fill represent the diminish'd Gravity. and answer to the Length of Pendulums at the Æquator, whilft g = c - l + m + n or g - c + l + m + n represents the diminish'd Gravity, and consequently the Length of Pendulums at *Paris*. If s be equal to l+m+ n, Pendulums will be as long at the Æquátor as at Paris; and if s be greater than l+m+n, Pendulums will be longer at the Æquator. But making all possible Allowance, in Favour of Mons. Mairan's Hy pothesis, no Calculation will bring l+m+n to be greater than, or ever equal to s. Therefore Monf. Mairan's Demonstrations, above-mentioned, are of no

And now, I think, I have answer'd all that relates to the Figure of the Earth in Mons. Mairan's Dissertation; in shewing, That his Conjectures can neither be supported by those Physical Principles which Sir Isaac Newton has Mathematically deduc'd from unquestioned

Force to prove the Earth to be an oblong Spheroid.

questioned Observations and Experiments accurately made; nor even by those Principles which He (M. Mairan) has assum'd to serve his intended Purpose—That his Demonstrations relating to the Difference of the A-Etion of the Centrifugal Force, are of no Service to him, for reconciling the Experiments made on Pendulums. with Monf. Cashini's Measures; -because, when applied to Sir Isaac Newton's Principles, they will make Pendulums longer at the Equator than at Paris, and when applied to Mons. Mairan's own Principles. they will make them a whole Inch shorter at the Æquator than at Paris, contrary to all Observations. which, at a Medium, make Pendulums but about two Lines or 166 of an Inch longer at the Equator than at Paris. - That he has built his Demonstrations upon a wrong Notion of Gravity - And that he has not consider'd what is most material in the Effect of the Centrifugal Force, acting on Bodies descending by their Gravity, between the Aguator and the Poles, namely, the Alteration of their Line of Direction, which would make them fallout of the Perpendicular towards the Aguator.

I shall add one more Philosophical Argument, given me by a Friend, to whom I communicated my Thoughts on this Subject; because it is wholly independent on those Principles of Philosophy, concerning which, some of the Gentlemen that believe the oblong spheroidical Figure of the Earth, and the English Philosophers, are

not yet agreed; and it is this.

If the Earth was of an oblong spheroidical Figure, higher at the Poles than the Æquator; the Axis of its Revolution, wou'd either go thro' one of its short Diameters, or be continually changing unless the said Axis did exactly coincide with the Axis of the Figure.

DE-

DEMONSTRATION.

Suppose fuch an oblong Figure as A a * fix'd to the Axis Pp at the Center C, but capable of moving freely round it towards P or towards p, yet so as to be oblig'd to move with the Axis, when it is turned round. pose now the Poles P and p to be fix'd, and the Body, thus constituted, to be turn'd swiftly round the Axis Pp; then if the Angle ACP be oblique, and the Figure A Da E be oblong, the Parts A Cand Ca will acquire a Centrifugal Force, which will enlarge the Angle p C A, till it comes to be a right one. Besides this, a Velocity will be generated in the Motion, while A is going towards the Perpendicular a C, which will make it go farther on towards P, as to B, with a Motion which will after that be retarded, till the Centrifugal Force has Strength enough to fend it back again the contrary Way; and so it will move continually with a reciprocal Motion, like the Oscillation of a Pendulum; and if a little of this Motion be lost at every Oscillation, then the oblong Figure ADaE will at last move quietly about its leffer Axis DE coinciding with Pp.

If An did not at first exactly coincide with Pp, the Centrifugal Force will have the above-mentioned Effect; and that this is not the Case in the Earth is more than probable, because the unequal Distribution of Sea and Land, besides the Phanomena of the Tides must make the Axis of its Gravity, and consequently the Axis of its Revolution, to differ from the Axis of the oblong Spheroid, if the Earth had such a Figure; without considering that every Earth-quake would alter so nice an Aquilibrium, which once lost,

wou'd never be recover'd again.

To leave nothing unexamin'd, relating to the Controversy. I have again consider'd the Measures and Observations, mentioned in the Account of the Meridian drawn thro' France, in the Memoirs of the Royal Academy. for the Year 1720; and I find them to want a great deal of the Accuracy requir'd in so nice a Point, as determining the different Lengths of Degrees upon the Sur-To prove my Affertion, I beg that face of the Earth. the Reader will examine the following Tables, whereby it appears, that if any thing certain can be deduc'd from the faid Observations and Measures, (either taken as they are, or reduc'd to the Level of the Sea, by the Rules given by Monf. Cassini*) it will be in Favour of Sir Isaac Newton's Figure of the Earth, rather than theirs.

In the following Table, the first Column gives the Names of Places; the second, the Distances from Paris, according to the Measures taken by the French Gentlemen; the third, the Latitudes observed by the same; the fourth, the Latitudes, such as the measur'd Distances will give them, supposing the Earth spherical; the fifth, the Differences between these and the Latitudes observed, express'd in Seconds of a Degree, where when the Latitude computed, exceeds the Latitude observed, the Letter N (North) shews that Difference to be in Favour of Mons. Cassini's Figure, and the contrary Difference mark'd by the Letter S (South) is in Favour of Sir Isaac Newton's Figure.

^{*} Memoirs for the Year 1720. Vol. I. P. 1. Ch. 13.

Names of Places.	Distances from Paris measur'd.	Latitudes ob- ferv'd.			Latitudes in a fpherical Earth computed from the measur'd Distances.			Differences in Seconds.	
I.	II.	III.			IV.			V.	
	Toises.			4.1					
Dunkirk.	125552	510	21	25112	510	21	25112	011	
Am iens.	60370	49	53	48	49	53	48	011	
Sourdon,	4997¢±	49	42	42	49	42	52,I	10,1 N	
Paris.		48	50	10	48	50	20,3	10,3 N	
Malvoisine.	18838	48	30	47	48	30	32,1	14,9 S	
Vouson.	67962	47	39	17	47	38	53,6	23,4 S	
Bourges.	100192	47	4	3 I	47	04	58,7	27,7 N	
S. Sauvier.	139934	46	23	24	46	23	12	12,0 S	
Croc.	169540	45	5 I	43	45	52	4,6	21,6 N	
Bort.	196484	45	23	27	45	23	45,2	18,2 N	
Aurillac.	223606	44	55	13	44	55	14,5	1,5 N	
Rodès.	256575	44	20	54	44	20	35,1	19,9 S	
Alby.	280612	43	55	32	43	55	19	13,0 S	
Carcassone.	321430	43	12	55	43	12	24,5	31,5 S	
Collioure.	360604	42	3 I	I 3 😤	42	3 1	13,8		

In this Table it is to be observed that there is an equal Number of Differences marked N (North) and S (South) and if the Differences on each Side be added together, there will be 89",4 on the North Side, and 114,"7 on the South: This last agrees best with Sir Isaac Newton's Figure, which must be supposed for the Correction of so great a Difference.

In the next Table, the first Column gives the Names of Places; the second, the Latitudes observed; the third, the Distances in the Meridian from *Paris*, reduced to the Level of the Sea; the fourth, the Differences of the second Column expressed in Seconds of a Degree; the fifth, the Differences of the Numbers in the third Column; and the sixth, the Measure of a Degree by the fourth and fifth Columns compared.

(302)							
I. Rames of Places.		11.		III.	IV. Seconds	V.	VI.
Dunkirk.	510	2.1	1911	Toifes.	of a De- gree.	Toiles.	Toiles,
Amiens. Clermont.	49 49	53	56	60444	4103 ¹ 1859	65010 29416	57040 56965
The R. Observatory.	48	50 39	10	67959	1967 4253	31028 67959	56787 57525
S. Sauvier.	46 45	23 51	24 43	139937	4553 1901	71978 29602	56912 56058
Bort. Aurillac.	45 4 4	23 55	46 13	196480 22361 6	1677	26941 27136	57834 57028
Rodés. Alby.	44 43	20 55	53 32	256474 280612	2060 1521	32858 24138	57422 57131
Carcaffone, Collioure.	43 42	12 31	55 13	321430 360614	2557 2502	40818 39184	57468 56380
							ļ

In this Table in the third Column, over-against St. Sauvier, the Number which was 139944 is corrected to make it 139937, to the Advantage of the oblong Figure. In the fixth Column, the Numbers appear fo irregular, as to be unfit to decide this Controversy. Then if a Comparison be made between Dunkirk, St. Sauvier (which is very near the Middle of France, and almost in the Meridian of Paris) and Collioure, the Measurement is absolutely in Favour of Sir Isaac Newton's Theory; the mean Degree between Dunkirk and St. Sauvier being larger by about 64 Toifes, than between S. Sauvier and Collioure; and to reduce them even to an Equality, there must be a greater Alteration made in the Situation of those three Places, than it is reasonable to suppose their Observations to be capable of admitting. Here follows the Comparison.

Dunkirk and Collioure		157061
Dunkirk and Paris		56960
Paris and Collioure	🖟 A mean Degree is -	57097
Dunkirk and S. Sauvier		57090,4
S. Sauvier and Collioure		57026,5
According	g to Monf. Picard,	57060
	2	To

To conclude, I will propose a Method of observing the Figure of the Shadow of the Earth in Lunar Eclipses, whereby the Difference between the Diameters in the oblong spheroidical Figure, if there be such an one as Mons. Cassini affirms (viz. of 96 to 95) may be discover'd.

Let PÆPÆ * represent the Earth, seen from the Sun at the Time of the Summer Solstice; it is evident, that the same Figure will express the Section of the Earth's Shadow at the Moon's Distance, as seen from the Earth. If EE represents the Ecsiptick, ÆÆ will be the shortest Diameter of the Section; and if LL be taken for the Moon's Way, in a total and central Eclipse of the Moon, by observing the Time which is spent in the Passage of the Center of the Moon, thro' the Shadow, and reducing that Time to Seconds of a Degree of a great Circle of the Heavens, we shall have the least Diameter of the Shadow.

Again, let the same Letters represent the same Things, only here the Section of the Shadow is such, as the Earth will cast at the Aquinox, and the Eclipse of the Moon is here supposed partial, its Center just touching the Shadow. When the Moon's Center is got to c, if the Latitude of its Center or its Distance from the Ecliptick be observed, we shall have the Length c C nearly equal to the longest Semi-diameter of the Shadow.

Now, comparing cC* in this Figure to LC* in the former (the Difference between cC and CP (Fig. 7.) and between CL and CÆ (Fig. 6.) not being worth notice) they ought to be to one another, as 96 to 95, which in such a Shadow will give a Difference of about 25" at a Medium, sensible enough to be observed, notwithstanding the Penumbra. If therefore those A-

^{*} Fig. VI. † Fig. VII. * Fig. VII. * Fig. VII. ftronomers

stronomers who have Instruments nice enough, and sufficient Skill in the Management of them, to take Angles to 3 or 4 Seconds of a Degree, will observe what I have been mentioning in total and partial Eclipses of the Moon; by such Observations they will easily convince us, that the Figure of the Earth is such as Mons. Cassini supposes it, or convince him that he has been misstaken.

The Semi-diameter of the Earth's Shadow, when the Earth is in Perihelio, and the Moon in Apogæo is 38', or 2280'', without considering the Encrease of the Shadow, on account of the Atmosphere of the Earth, which wou'd make it 39' or 2340" (allowing one Second for a Mile;) and the Semi-diameter of the Shadow, when the Earth is in Aphelio, and the Moon in Perigao is 461, 20", or 278011, which encreased on account of the Atmosphere of the Earth, will bring it to 47', 20" or 2840". Now if the Proportion of 95 to 96 be taken in both Cases you will have these Analogies, $\begin{cases} 95:96::2340'':2364''.6 \\ 95:96::2840'':2869'',8 \end{cases}$ So that 2364'',6- 2340" = 24" 6 will be the Difference of the Semi-diameters, when the Section of the Shadow is the leaft, and 2869''.8 - 2840'' = 29''.8 will be the Difference of Semi-diameters, when the Section of the Shadow is the greatest; the Sum of those Differences 24",6 + 29",8 halved, will give the Difference, when the Section of the Shadow is at a Medium = 27" 4; from which if we take 211,4 because in Fig. 7. Cc is a little less than CP, and in Fig. 6. LC is something greater than A.C., we shall have C c in Fig. 7. to compare with LC in Fig. 6. which will exceed it by 25", if Mons. Cassini's Figure of the Earth be the true one.